that the deviation starts at the rear part of the airfoil where the gas velocity is higher.

Conclusion

The theoretical method developed here gives good agreement with experiments for different flow conditions in the incompressible and compressible flows. This theoretical method provides the designer with a means of estimating the forces on the blade in cascades. It has been observed that the compressibility effects are practically insignificant. This effect increases, however, with increasing α and is more pronounced at the rear part of the blade.

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Position of the Thrust Line and Longitudinal Stability

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Nomenclature†

A = aspect ratio
c = mean chord
C_D = $D/(qS)$ = drag coefficient
C_L = $L/(qS)$ = lift coefficient
C_M = $M/(qSc)$ = moment coefficient
C_{mu} = $(1/Sqc)(V\partial/\partial V)M_{c.g.} \approx -2C_D z_T$
$C_{m\alpha}$ = $(1/Sqc)(\partial/\partial\alpha)M_{c.g.} \approx s dC_L/d\alpha$
C_{xu} = $(1/Sq)(V\partial/\partial V)F_x \approx -2C_D$
$C_{x\alpha}$ = $(1/Sq)(\partial/\partial\alpha)F_x \approx C_L(1 - 2dC_L/d\alpha/\pi e A)$
C_{zu} = $(1/Sq)(V\partial/\partial V)F_z \approx -2C_L$
$C_{z\alpha}$ = $(1/Sq)(\partial/\partial\alpha)F_z \approx -dC_L/d_\alpha$
D = drag
e = efficiency factor in $C_D = C_{DO} + C_L{}^2/(\pi e A)$
F_x = $T - D$ = horizontal force
F_z = $mg - L$ = vertical force
g = acceleration in free fall
L = lift
M = aerodynamic moment
$M_{c.g.}$ = $M + Tz_Tc$ = over-all moment
m = mass of the airplane
q = $\frac{1}{2}\rho V^2$ = dynamic pressure
s = static margin (negative) expressed as a fraction of the chord
S = wing area
T = thrust
V = true airspeed
z_T = vertical displacement of the thrust line (positive downward) expressed as a fraction of the chord
α = angle of attack
γ = flight-path angle
δ = denotes a small variation in a quantity
ζ = damping ratio
η = L/D
ω = angular frequency

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† Time derivatives are denoted by dots over letters.

Introduction

The phugoid motion is usually assumed to take place at a constant angle of attack.[1] The present Note points out that this is only true so long as the thrust line is not appreciably displaced from the airplane c.g. Expressions for the frequency and damping of the phugoid, which are valid when a displacement of the thrust line does occur, are presented.

The effect of a displaced thrust line on the frequency is easily appreciated qualitatively. Suppose the thrust line is below the c.g. In this case there exists a nose up thrust moment that is balanced in straight and level flight by a nose down aerodynamic moment. The two moments depend differently on airspeed, therefore a change of airspeed disturbs the balance. As the airplane noses down and picks up speed, the nose down aerodynamic moment overcomes the nose up thrust moment and tends to push the nose further down. This cuts down the "restoring force" and with it the frequency of the phugoid. This qualitative argument is borne out by the quantitative analysis below. The analysis shows that the damping of the phugoid is also reduced. It is found that a large displacement of the thrust line below the c.g. results in unstable phugoid oscillations and eventually in nonoscillatory divergence.

The analysis is carried out explicitly for a conventional low-speed airplane with the variation of thrust with airspeed neglected. Results of a more general analysis applicable to a general airplane are also presented.

Equations of Motion

The equations governing the phugoid motion are

$$mV\dot{\gamma} = L - mg\cos\gamma \tag{1}$$

$$m\dot{V} = T - D - mg\sin\gamma \tag{2}$$

$$0 = M + Tz_Tc \tag{3}$$

The last equation expresses the balancing of moments. It is assumed that a short period response of high frequency and damping makes sure that the moments are always very nearly balanced and angular accelerations may be neglected.

Consider a steady straight and level flight condition in which

$$\gamma = 0 \tag{4}$$

$$L = mg \tag{5}$$

$$D = T = \eta^{-1}mg \tag{6}$$

$$M = -Tz_Tc \tag{7}$$

When Eqs. (1-3) are linearized around this condition, one finds

$$mV\delta\dot{\gamma} = \delta L \tag{8}$$

$$m\delta\dot{V} = \delta T - \delta D - mg\delta\gamma \tag{9}$$

$$0 = \delta M + \delta T z_T c \tag{10}$$

Assume a low-speed airplane with thrust independent of airspeed

$$T = \text{const} \quad (11)$$

$$C_D = C_{DO} + C_L^2/(\pi e A) \quad (12)$$

$$C_M = C_{MO} + sC_L \quad (13)$$

Then

$$\delta T = 0 \quad (14)$$

$$\delta L = L(\delta C_L/C_L + 2\delta V/V) = mg(\delta C_L/C_L + 2\delta V/V) \quad (15)$$

$$\delta D = D(\delta C_D/C_D + 2\delta V/V) = \eta^{-1}mg2\delta C_L/(\pi e A) + 2\delta V/V) \quad (16)$$

$$M = M(\delta C_M/C_M + 2\delta V/V) = scmg\delta C_L/C_L - z_T c\eta^{-1}mg2\delta V/V \quad (17)$$

Equation (10) enforces $\delta M = 0$ which in turn implies

$$\delta C_L/C_L = (2/\eta)(z_T/s) \quad (18)$$

This entails

$$\delta L = 2mg[1 + \eta^{-1}(z_T/s)]\delta V/V \quad (19)$$

$$\delta D = 2\eta^{-1}mg[1 + 2(z_T/s)C_L/(\pi e A)]\delta V/V \quad (20)$$

When these are substituted in Eqs. (8) and (9), the equations of motion for δV and $\delta\gamma$ emerge as

$$\delta\dot{\gamma} = (2g/V^2)[1 + \eta^{-1}(z_T/s)]\delta V \quad (21)$$

$$\delta\dot{V} = -2\eta^{-1}(g/V)[1 + 2(z_T/s)C_L/(\pi e A)]\delta V - g\delta\gamma \quad (22)$$

Finally, δV may be solved for from Eq. (21) and substituted in Eq. (22). This results in

$$\delta\ddot{\gamma} + 2\eta^{-1}(g/V)[1 + 2(z_T/s)C_L/(\pi e A)]\delta\dot{\gamma} + 2(g/V)^2[1 + \eta^{-1}(z_T/s)]\delta\gamma = 0 \quad (23)$$

From this last equation the frequency and damping of the phugoid may be read off as

$$\omega = 2^{1/2}(g/V)(1 + \eta^{-1}z_T/s)^{1/2} \quad (24)$$

$$\zeta = (1/2^{1/2}\eta)(1 + \eta^{-1}z_T/s)^{-1/2}[1 + (2C_L/\pi e A)z_T/s] \quad (25)$$

Discussion

In Eqs. (24) and (25), the first factor on the right is the usual expression for the phugoid with $z_T = 0$. The other factors are corrections due to z_T. It is seen that the pertinent parameter is z_T/s. The static margin s is negative for a statically stable airplane. A positive z_T (i.e., thrust line below the c.g.) results in lowering the frequency and damping ratio of the phugoid.

For excessively large negative values of z_T/s, the factors on the right-hand side of Eqs. (24) and (25) may reach zero. When the last factor in Eq. (25) reaches zero, the oscillations become undamped. Further increase of z_T makes the damping negative and leads to divergent oscillations. When the last factor in Eq. (24) becomes small, the natural frequency decreases. At the same time $|\zeta|$ which is proportional to the inverse of this factor increases. Eventually $|\zeta|$ goes through 1 and exponential rather than oscillatory behavior results. If this happens after ζ has become negative, the phugoid motion is changed into exponential divergence. These effects are enhanced by small η and by high C_L.

A General Airplane

For a general airplane the variations in lift, drag, thrust, and aerodynamic moment are subject to various effects, including dependence on Mach number, and explicit expressions are not available. The analysis just presented may still be performed in terms of stability derivatives. When this is done, Eqs. (24) and (25) are replaced by

$$\omega = (-C_{zu}/C_L)^{1/2}(g/V)[1 - (C_{z\alpha}/C_{zu})C_{mu}/C_{m\alpha}]^{1/2} \quad (26)$$

$$\zeta = -\frac{\frac{1}{2}C_{xu}}{(-C_L C_{zu})^{1/2}}\left(1 - \frac{C_{z\alpha}}{C_{zu}}\frac{C_{mu}}{C_{m\alpha}}\right)^{-1/2}\left(1 - \frac{C_{x\alpha} - C_L}{C_{xu}}\frac{C_{mu}}{C_{m\alpha}}\right) \quad (27)$$

It is seen that the parameter that takes the role of z_T/s is

$$C_{mu}/C_{m\alpha}$$

The thrust line displacement z_T manifests itself in C_{mu} which for the simple airplane of the last section becomes $-2C_D z_T$. The stability derivative C_{mu} is most often neglected.[1] It should be realized that the smallness of C_{mu} is vital to the conventional analysis of the longitudinal modes. If C_{mu} is appreciable, the angle of attack is no longer decoupled from the phugoid motion and the phenomena described above result. Also the airspeed is no longer decoupled from the short period mode. This last point, however, is outside the scope of the present Note.

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Vortex Flow over Helicopter Rotor Tips

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Introduction

As part of a recent study into the boundary-layer flow on helicopter rotor blade tips at The Ohio State University, flow visualization tests were conducted to reveal local flow direction at many points on various shaped tips. One particularly interesting set of flow traces was obtained with a square tip configuration. This Note presents some of the highlights of that study.

Flow Visualization Technique

The flow over the tips was studied by using a flow visualization technique whereby ammonia vapor was expelled (over a short time duration) from a network of orifices in the tip and carried by the boundary-layer flow over a diazonium salt

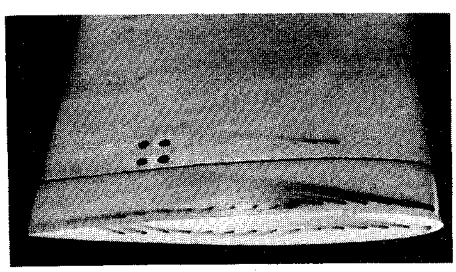

Fig. 1 Top view of square tip at 400 rpm and 10° pitch angle.

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